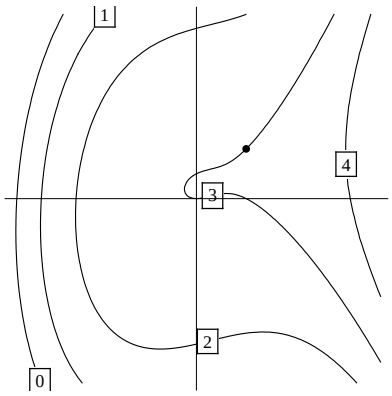
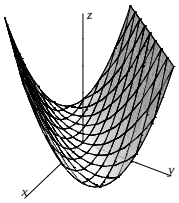
Directional Derivatives

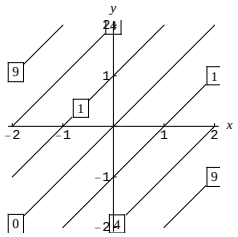
1. Here is the level set diagram of a function $f(x, y)$; the value of f on each level set is labeled. Imagine that $f(x, y)$ represents temperature on the blackboard, and an ant is standing at the point (a, b), which is marked on the diagram.



   - What direction should the ant go to warm up most quickly? That is, in what direction should he go to experience the highest instantaneous rate of change of temperature (with respect to distance)?
   - What direction should the ant go to cool down most quickly? That is, in what direction should he go to experience the lowest (most negative) instantaneous rate of change of temperature?

2. Let $f(x, y) = (x - y)^2 = x^2 - 2xy + y^2$. (The graph and level set diagram of f are shown.)





   Calculate the following directional derivatives of f.

   (a) $D_{\vec{u}} f(1, 0)$ where $\vec{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$.

   (b) $D_{\vec{u}} f(1, 0)$ where $\vec{u} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$.

   (c) $D_{\vec{u}} f(0, 1)$ where $\vec{u} = \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$.

   (d) $D_{\vec{u}} f(0, 1)$ where $\vec{u} = \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$.

3. A fly is flying around a room in which the temperature is given by $T(x, y, z) = x^2 + y^4 + 2z^2$. The fly is at the point $(1, 1, 1)$ and realizes that he's cold. In what direction should he fly to warm up most quickly? If he flies in this direction, what will be the instantaneous rate of change of his temperature?

4. You're hiking a mountain which is the graph of $f(x, y) = 15 - x^2 - 2xy - 3y^2$. You're standing at $(1, 1, 9)$. You wish to head in a direction which will maintain your elevation (so you want the instantaneous change in your elevation to be 0). How many possible directions are there for you to head? What are they?